Straight Line Motion Revisited

Warm-up

1. Given the position function $s(t)=4t^3-10t+2$, find the velocity and acceleration functions.

$v(t)=12t^2-10$ $a(t)=24t$

2. Summarize the relationships between the position function, $s(t)$, the velocity function, $v(t)$, and the acceleration function, $a(t)$.

$v(t)=s'(t)$ $a(t)=v'(t)$

3. Imagine that a man starts walking 5 miles due north, then turns around and walks 3 miles due south.

distance a) What is the total distance that he walked? 8 miles (direction doesn't matter)

displacement b) How far away from his initial position (and what direction) does he end up?

$+5 + -3 =$ 2 miles north of where he started

north 5 south 3

Finding Position and Velocity by Integration

If we know the position function, we can find the velocity function by differentiating

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If we know the velocity function, we can find the acceleration function by differentiating

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Summary

① $\int v(t)\,dt = s(t)$ ② $\int a(t)\,dt = v(t)$

Example 1: Suppose that a particle moves with velocity $v(t)=\cos \pi t$ along a straight line. Assuming that the particle has coordinate $s=4$ at time $t=0$, find its position function.

Formula 1: $\frac{1}{\pi}\int \cos \pi t \, dt(\pi) = s(t)$

u-sub: $u=\pi t$
$du=\pi\,dt$

$s(t)=\frac{1}{\pi}\int \cos u\,du = \frac{1}{\pi}\sin u + C$

$\rightarrow s(t) = \frac{1}{\pi}\sin \pi t + C$

$4 = \frac{1}{\pi}\sin(\pi \cdot 0) + C$

$4 = \frac{1}{\pi}(0) + C$

$C = 4$

$\boxed{s(t)=\frac{1}{\pi}\sin \pi t + 4}$

s
c
-s
-c
s

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Practice Problem 1: Suppose that a particle is moving along a straight line with velocity $v(t) = 2t + 1$. If at time $t = 0$ the particle is at position $s = 2$, find its position function.

$s(t) = \int (2t+1)\,dt$

$s(t) = \frac{\cancel{2}t^2}{\cancel{2}} + t + C$

$s(t) = t^2 + t + C \rightarrow \boxed{s(t) = t^2 + t + 2}$

$2 = 0^2 + 0 + C$

$2 = C$

Displacement versus Distance

$$\text{Displacement} = \int_{t_0}^{t_1} v(t)\,dt$$

(New position relative to original position)

$$\text{Distance} = \int_{t_0}^{t_1} |v(t)\,dt|$$

Direction doesn't matter → abs. value

Example 2: Suppose that a particle moves on a straight line so that its velocity at time t is $v(t) = t^2 - 2t$ meters/second.

a) Find the displacement of the particle over [0, 3].

b) Find the distance traveled by the particle over [0, 3].

① Rewrite

$|t^2 - 2t| = \begin{cases} t^2 - 2t & \text{if } t^2 - 2t \geq 0 \\ -t^2 + 2t & \text{if } t^2 - 2t < 0 \end{cases}$

$t^2 - 2t = 0$

$t(t-2) = 0$

$t = 0 \quad t = 2$

0 2

$\begin{cases} t^2 - 2t & \text{if } t \geq 2 \\ -t^2 + 2t & \text{if } t < 2 \end{cases}$

a) $\text{Displ.} = \int_0^3 (t^2 - 2t)\,dt$

$= \left[\frac{t^3}{3} - \frac{\cancel{2}t^2}{\cancel{2}}\right]_0^3$

$(9 - 9) - (0)$

$\boxed{0 \text{ meters from start}}$

b) $\text{Dist} = \int_0^3 |t^2 - 2t|\,dt$

② $\int_0^2 (-t^2 + 2t)\,dt + \int_2^3 (t^2 - 2t)\,dt$

③ $\left[-\frac{t^3}{3} + \frac{\cancel{2}t^2}{\cancel{2}}\right]_0^2 + \left[\frac{t^3}{3} - \frac{\cancel{2}t^2}{\cancel{2}}\right]_2^3$

$(-\frac{8}{3} + 4) - (0) + (\frac{27}{3} - 9) - (\frac{8}{3} - 4) = \frac{11}{3} - 1 = \boxed{\frac{8}{3} \text{ meters}}$

Practice Problem 2: A particle is moving so that its velocity, $v(t) = 8 - 2t$ over [0, 5]. Find the displacement and distance travelled by the particle.

$\text{Displ} = \int_0^5 (8 - 2t)\,dt$

$= \left[8t - \frac{\cancel{2}t^2}{\cancel{2}}\right]_0^5 = (40 - 25) - (0) = \boxed{15\,M}$

$\text{Dist} = \int_0^5 |8 - 2t|\,dt$

① Rewrite abs val as piecewise:

$|8 - 2t| = \begin{cases} 8 - 2t & \text{if } 8 - 2t \geq 0 \\ -8 + 2t & \text{if } 8 - 2t < 0 \end{cases}$

$\boxed{\begin{cases} +8 - 2t & \text{if } t \leq 4 \\ -8 + 2t & \text{if } t > 4 \end{cases}}$

$8 - 2t = 0$

$8 = 2t$

$t = 4$

0 4

② $\int_0^4 (+8 - 2t)\,dt + \int_4^5 (-8 + 2t)\,dt$

$\left[8t - \frac{\cancel{2}t^2}{\cancel{2}}\right]_0^4 + \left[-8t + \frac{\cancel{2}t^2}{\cancel{2}}\right]_4^5$

$(32 - 16) - (0) + (-40 + 25) - (-32 + 16)$

$\boxed{17 \text{ meters}}$

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Practice Problems

1. A particle is moving so that its velocity, $v(t) = 4 - t$ over [0, 6]. Find the displacement and distance travelled by the particle.

DISPLACEMENT

displ $= \int_0^6 (4-t)\,dt$

$\left[4t - \frac{t^2}{2}\right]_0^6$

$(24-18)-(0)$

$= \boxed{6}$

DISTANCE: $\int_0^6 |4-t|\,dt$ = ② $\int_0^4 (4-t)\,dt + \int_4^6 (-4+t)\,dt$

① $|4-t| = \begin{cases} 4-t & \text{if } 4-t \ge 0 \\ -4+t & \text{if } 4-t < 0 \end{cases}$

$4-t=0$

$t=4$

$= \boxed{\begin{cases} 4-t & \text{if } t \le 4 \\ -4+t & \text{if } t > 4 \end{cases}}$

$\left[4t-\frac{t^2}{2}\right]_0^4 + \left[-4t+\frac{t^2}{2}\right]_4^6$

$(16-8)-(0)+(-24+18)-(-16+8)$

$\boxed{10}$

2. A particle is moving so that its velocity, $v(t) = t^2 - t - 2$ over [0, 3]. Find the displacement and distance travelled by the particle.

DISPLACEMENT

disp $= \int_0^3 (t^2-t-2)\,dt$

$\left[\frac{t^3}{3} - \frac{t^2}{2} - 2t\right]_0^3$

$\left(9-\frac{9}{2}-6\right)-(0)$

$\frac{18-9-12}{2} = \boxed{-\frac{3}{2}}$

DISTANCE: $\int_0^3 |t^2-t-2|\,dt$ = ② $\int_0^2 (-t^2+t+2)\,dt + \int_2^3 (t^2-t-2)\,dt$

① $|t^2-t-2| = \begin{cases} t^2-t-2 & \text{if } t^2-t-2 \ge 0 \\ -t^2+t+2 & \text{if } t^2-t-2 < 0 \end{cases}$

$t^2-t-2=0$

$(t-2)(t+1)=0$

$t=2$ $\cancel{t=-1}$

$\boxed{\begin{cases} t^2-t-2 & \text{if } t \ge 2 \\ -t^2+t+2 & \text{if } t < 2 \end{cases}}$

$\left[-\frac{t^3}{3}+\frac{t^2}{2}+2t\right]_0^2 + \left[\frac{t^3}{3}-\frac{t^2}{2}-2t\right]_2^3$

$\left(-\frac{8}{3}+2+4\right)-(0)+\left(\frac{27}{3}-\frac{9}{2}-6\right)-\left(\frac{8}{3}-2-4\right)$

$\frac{11}{3} - \frac{9}{2} + 6 =$

$\frac{22-27+36}{6} = \boxed{\frac{31}{6}}$

3. Find the position, velocity, speed, and acceleration at time t = 1second of a particle if $v(t) = 2t - 4$; $s = 3$ when $t = 0$.

velocity

$v(t) = 2t-4$

$v(1) = 2-4 = \boxed{-2}$

speed $= |v(t)| = |-2| = \boxed{2}$

accel

$a(t) = v'(t)$

$a(t) = \boxed{2}$

position

$s(t) = \int v(t)\,dt$

$s(t) = \int (2t-4)\,dt$

$s(t) = \frac{2t^2}{2} - 4t + C$

$3 = 0^2 - 4(0) + C$

$3 = C$

$s(t) = t^2-4t+3$

$s(1) = 1-4+3 = \boxed{0}$